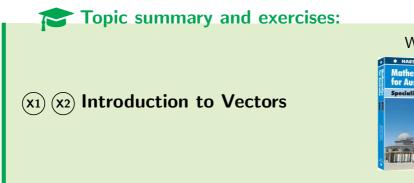


NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS EXTENSION 1 YEAR 12 COURSE





Name:

Initial version by H. Lam, September 2019 with assistance from I. Ham. Last updated April 22, 2024 Various corrections by students & members of the Mathematics Department at Normanhurst Boys High School.

Acknowledgements Pictograms in this document are a derivative of the work originally by Freepik at http://www.flaticon.com, used under 🕲 CC BY 2.0.

Parts of this handout are derived from Haese, Haese, and Humphries (2015),

MATH1131 Mathematics 1A and MATH1141 Higher Mathematics 1A Algebra Notes (2018), So and Wong (1987) and Pender, Sadler, Ward, Dorofaeff, and Shea (2019). Most GeoGebra applets listed here are courtesy of the NSW Department of Education's Mathematics Curriculum Support team/mEsh project's efforts.

Symbols used

A

k'n

Beware! Heed warning.

- Provided on NESA Reference Sheet
- Facts/formulae to memorise.
- (x1) Mathematics Extension 1 content.
- Literacy: note new word/phrase.

Further reading/exercises to enrich your understanding and application of this topic.

Syllabus specified content

- Facts/formulae to understand, as opposed to blatant memorisation.
- $\mathbb N~$ the set of natural numbers
- ${\mathbb Z}~$ the set of integers
- ${\mathbb Q}~$ the set of rational numbers
- ${\mathbb R}\,$ the set of real numbers
- $\forall \ \, \text{for all} \quad$



- For a thorough understanding of the topic, *every* question in this handout is to be completed!
- Additional questions from *Haese Mathematics for Australia Specialist Mathematics 11* (Haese et al., 2015) *CambridgeMATHS Year 12 Extension 1* (Pender et al., 2019) will be completed at the discretion of your teacher.

• Remember to copy the question into your exercise book!

Syllabus outcomes addressed ME12-2 applies concepts and techniques involving

ME12-2 applies concepts and techniques involving vectors and projectiles to solve problems

Syllabus subtopics

ME-V1 (1.1, 1.2) Introduction to Vectors (Introduction to vectors, Further operations with vectors)

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Section 1

Introduction to Vectors

1.1 **Definition**

Definition 1

Quantities which only have magnitude are known as *scalars*.

Example: speed of an aeroplane.

Definition 2

Quantities which have both <u>magnitude</u> and <u>direction</u> are known as *vectors*.

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Example: velocity of an aeroplane.

Fill in the spaces

Other examples of vector forces:

- acceleration force
- displacement

• momentum

1.2 **Representation**

1.2.1 Geometric

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Fill in the spaces

Vector quantities can be represented using a *directed line segment* or arrow.

- The length of the arrow represents the magnitude
- The arrowhead shows its direction.

Definition 3

Corollary 1

Example 1

Given \overrightarrow{PQ} as shown, draw two other vectors, $\underline{a} = \overrightarrow{AB}$ and $\underline{e} = \overrightarrow{EF}$ such that

$$\overrightarrow{PQ} = \overrightarrow{AB} = \overrightarrow{EF}$$

$$\overrightarrow{PQ} = \overrightarrow{Q}$$

1.2.2 Algebraic

From Example 1, there are three ways of writing the vector drawn:

P <

- \overrightarrow{PQ} : the vector commencing from the point P and ending at point Q.
- \vec{p} or p: NESA syllabus documentation, handwritten.

1.3 Vector arithmetic

1.3.1 Addition

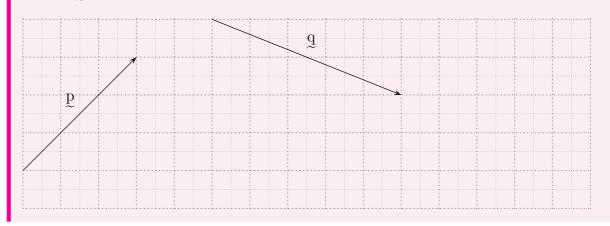


A Use a ruler!

Steps

For vectors **p** and **q**, to perform $\mathbf{p} + \mathbf{q}$:

- **1.** Draw p.
- **2.** Draw q, starting from the arrowhead of p.
- **3.** Draw resultant vector from the tail of \underline{p} to the head of q.



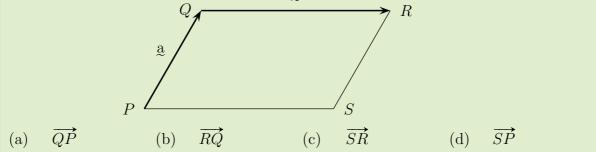
1.3.2 Subtraction/negative vectors

Definition 4

If \underline{p} is a vector, then $-\underline{p}$ is equal in $\underbrace{magnitude}_{magnitude}$ but $\underbrace{opposite}_{magnitude}$ in $\underbrace{direction}_{magnitude}$

Example 2

(Haese et al., 2015) PQRS is a parallelogram where $\overrightarrow{PQ} = \underline{a}$ and $\overrightarrow{QR} = \underline{b}$. Find vector expressions for: \underline{b}



1.3.3 Magnitude and zero vector

Definition 5

The *magnitude* of a vector \underline{p} is given by $|\underline{p}|$.

Definition 6

The zero vector has magnitude zero, i.e. $\left| \underline{\mathbf{p}} \right| = 0$. Direction is not defined.....

Usually written as $\underline{0}$ (handwritten: $\underline{0}$).

1.3.4 Scalar multiplication

Definition 7

If p is a vector, and $\lambda \in \mathbb{R}$,

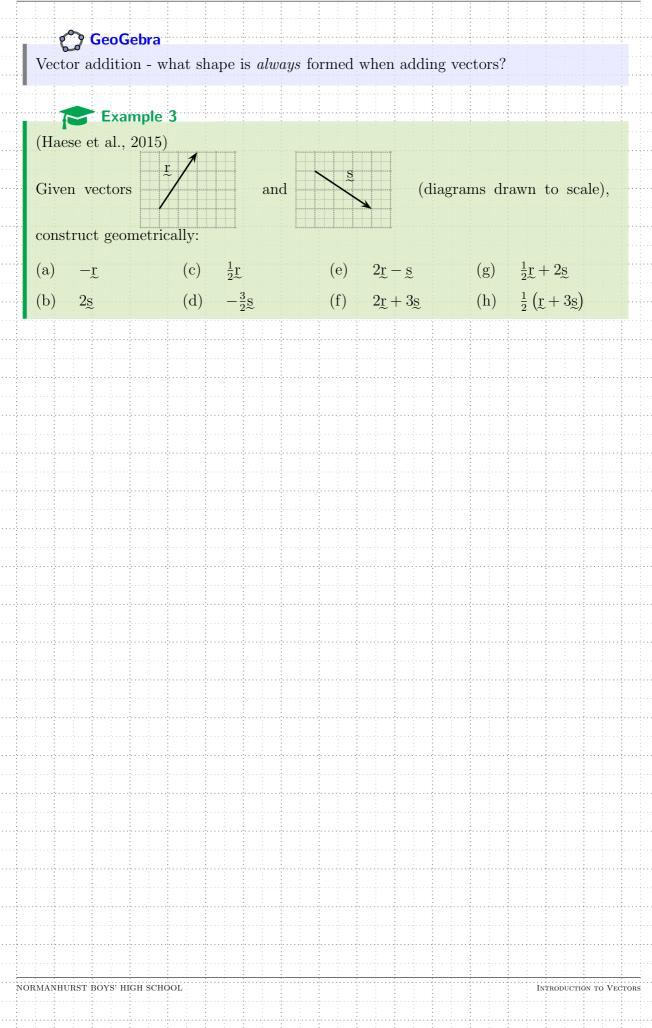
- If $\lambda > 1$, then $\lambda \underline{p}$ extends the <u>magnitude</u> to $\lambda |\underline{p}|$.
 - (<u>Extends</u> the vector \underline{p} by a factor of λ).
- If $0 < \lambda < 1$, then $\lambda \underline{p}$ shrinks the magnitude.
- If $\lambda = 0$, then $\lambda \underline{p} = \underline{0}$.
- If $\lambda < 0$, then $\lambda \underline{p}$ sees the magnitude scaled by $|\lambda|$ and direction reverses .

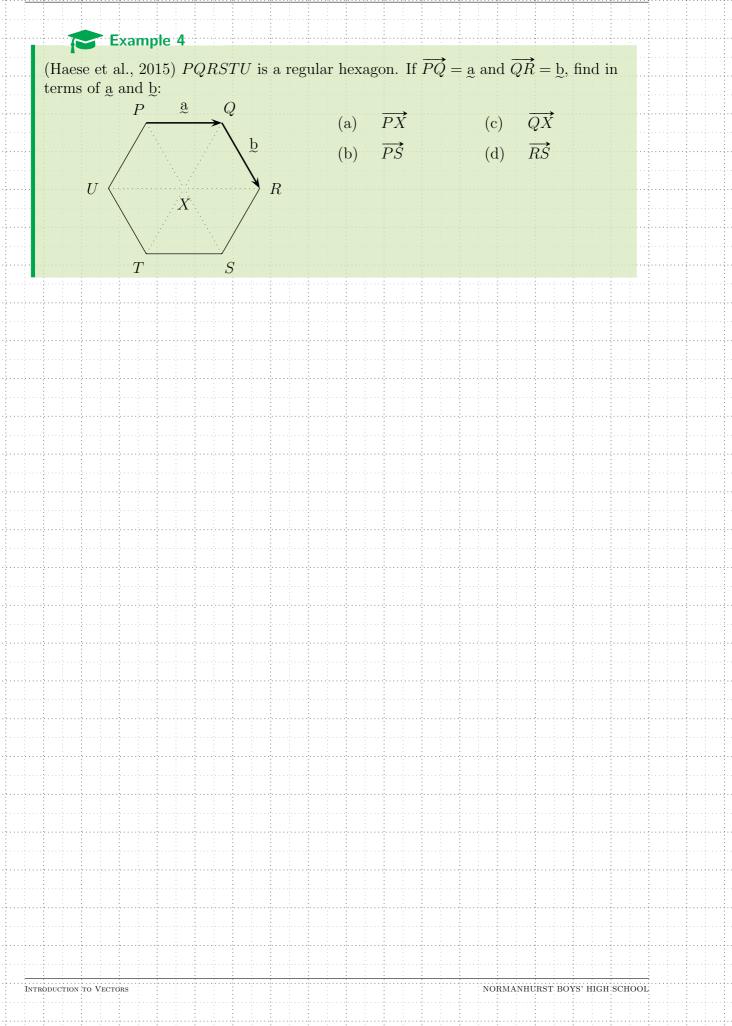
Laws/Results

- \underline{p} and $\lambda \underline{p}$, $\lambda > 0$ are <u>parallel</u>.
- \underline{p} and $\lambda \underline{p}$, $\lambda < 0$ are <u>antiparallel</u>.

🕎 GeoGebra

Scaling vector investigation





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1.4 Vector equations

Laws/Results

Vectors can be • Added

• Have scalar multiplication applied

in the same manner as usual scalar numbers, with the same commutative/associative/distributive laws.

Example 5

(MATH1131 Mathematics 1A and MATH1141 Higher Mathematics 1A Algebra Notes, 2018) Fully simplify: $3(2\underline{a} - \underline{b}) + (\underline{a} - 2\underline{b})$.

Example 6

(MATH1131 Mathematics 1A and MATH1141 Higher Mathematics 1A Algebra Notes, 2018) Fully simplify: $2\overrightarrow{AC} - \overrightarrow{OC} + \overrightarrow{OA}$.

Important note

- Draw picture if it's slightly uncertain what is \overrightarrow{AC} .
- Convert the vector arrow notation \overrightarrow{OC} to the single variable notation \underline{c} .

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Section 2

Simple Vector Geometry

2.1 Vector components

2.1.1 Unit vector

Definition 8

The *unit vector* has magnitude 1.

Laws/Results

For every vector \underline{p} there are two *normalised* (corresponding unit) vectors:

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- $\widehat{\mathbf{p}} = \frac{\mathbf{p}}{\left|\mathbf{p}\right|}$. Parallel to \mathbf{p} .
- $-\widehat{\mathbf{p}} = -\frac{\mathbf{p}}{\left|\mathbf{p}\right|}$. Antiparallel to \mathbf{p} .

🕜 GeoGebra Unit vector

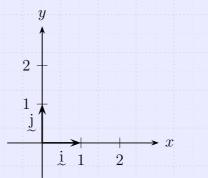
2.1.2 Cartesian basis vectors/components

Definition 9

In the Cartesian plane, define the *basis vectors* to be unit vectors in the

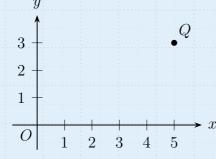
• x direction to be <u>i</u>

• y direction to be <u>j</u>



Fill in the spaces

• The position (5,3) can now be represented via a *translation vector*.



• Component notation:

$$QQ = 5i + 3j$$

• Column vector notation:

$$\binom{5}{3} = 5\binom{1}{0} + 3\binom{0}{1}$$

Definition 10

The point Q(x, y) has position vector

$$\overrightarrow{OQ} = \begin{pmatrix} x \\ y \end{pmatrix} = x \mathbf{i} + y \mathbf{j}$$

where

- The component form is $x \mathbf{i} + y \mathbf{j}$
- Represented in *column vector* notation: $\begin{pmatrix} x \\ y \end{pmatrix}$.
- Year 7-10 ordered pair notation: (x, y).

Important note

Pender et al. (2019) and *Mathematics Extension 1 Stage 6 Syllabus* (2017, Revised 18/11/2019) both use brackets instead of parentheses for the column vector delimiters, e.g.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Tertiary education providers however, tend to use parentheses.

Example 8

A ship S leaves port O and sails north-east at 20 km/h. Describe its position after 6 hours by giving its position vector as a sum of multiples of the basis vectors \underline{i} and \underline{j} .

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2.1.3 Position vs displacement vectors

Definition 11

A position vector of a point P is the vector \overrightarrow{OP} , where the vector commences at the origin and finishes at P.

A position vector gives the <u>absolute</u> <u>location</u> of a point in the plane.

• Each point P in the Cartesian plane corresponds to the position vector \overrightarrow{OP}

Definition 12

A displacement vector is the vector \overrightarrow{AB} , where the vector commences at the point A and finishes at B.

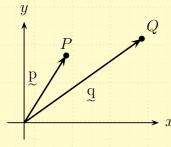
A displacement vector gives the relative position of a point from another.

• Vector of point A relative to point B: \overrightarrow{BA}

☐ Position Vector Investigation - see how a position vector is different to a displacement vector.

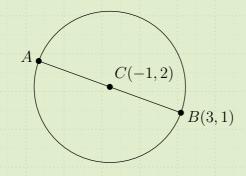
Laws/Results

For two points P and Q with position vectors $\underline{\mathbf{p}} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$ and $\underline{\mathbf{q}} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$ respectively,

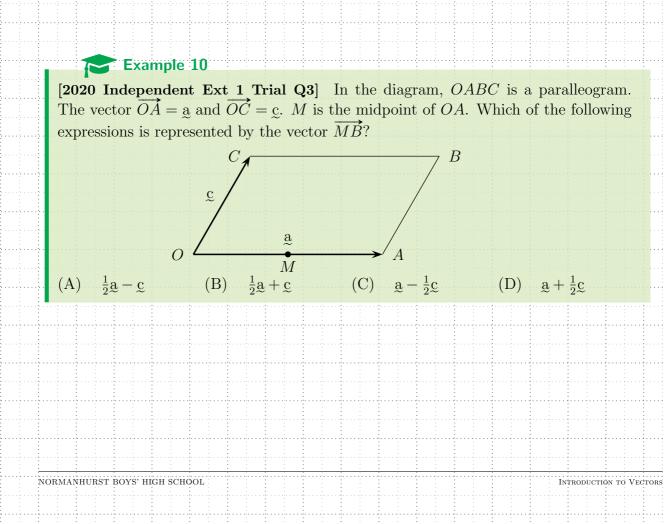


$\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$	$\overrightarrow{OQ} + \overrightarrow{QP} = \overrightarrow{OP}$
$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$	$\overrightarrow{QP} = \overrightarrow{OP} - \overrightarrow{OQ}$
$= \tilde{q} - \tilde{p}$	$=$ $\mathbf{p} - \mathbf{q}$
$= \begin{pmatrix} q_1 - p_1 \\ q_2 - p_2 \end{pmatrix}$	$= \begin{pmatrix} p_1 - q_1 \\ p_2 - q_2 \end{pmatrix}$
$= \left(q_2 - p_2\right)$	$-(p_2-q_2)$

Example 9 (Haese et al., 2015) AB is a diameter of a circle with centre C(-1, 2). If B is (3, 1), find:

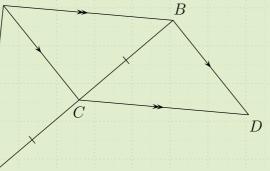


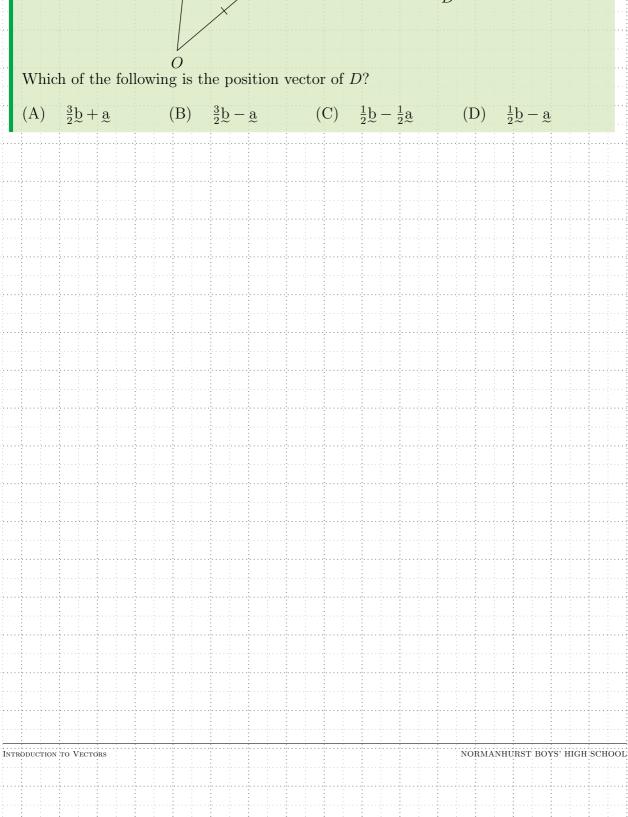
(a) *BC*(b) the coordinates of *A*.





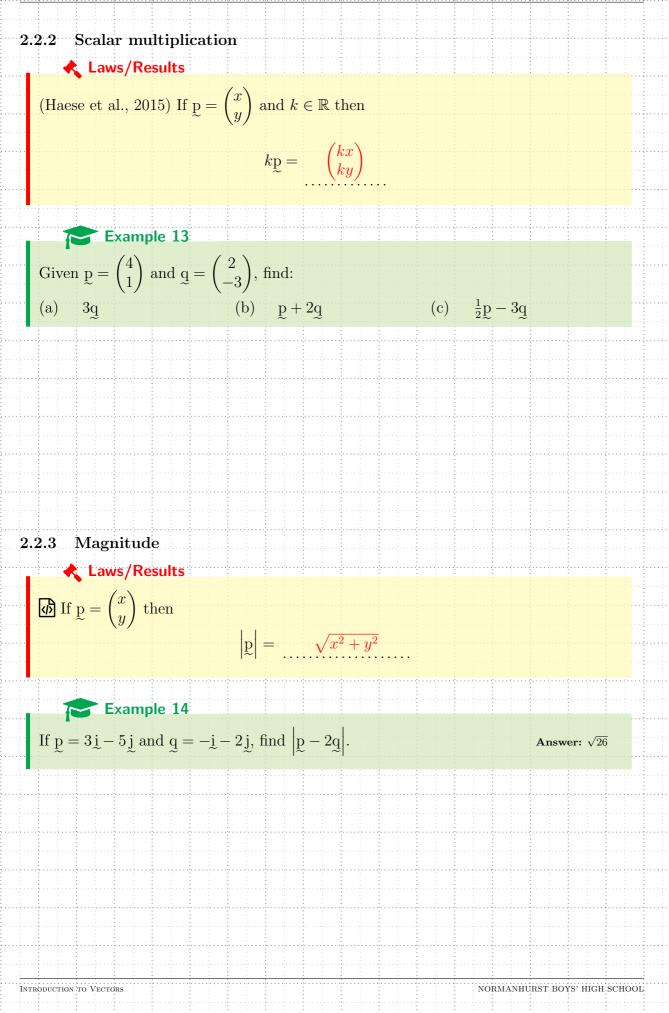
[2020 NEAP Ext 1 Trial Q7] The position vectors of points A and B are \underline{a} and \underline{b} respectively. Point C is the midpoint of OB and point D is such that ABCD is a parallelogram.

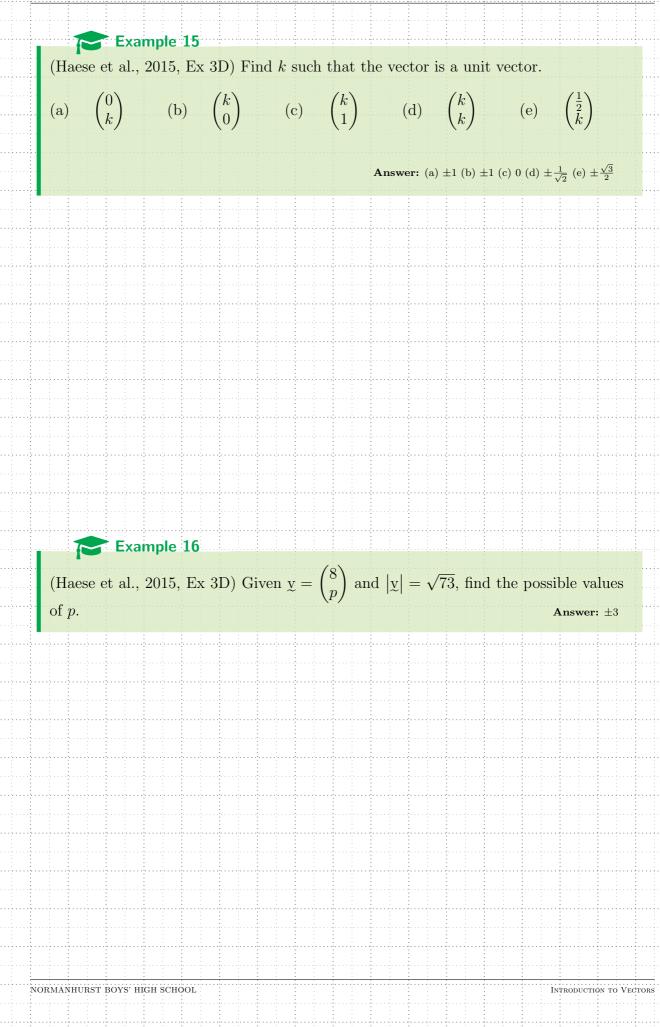


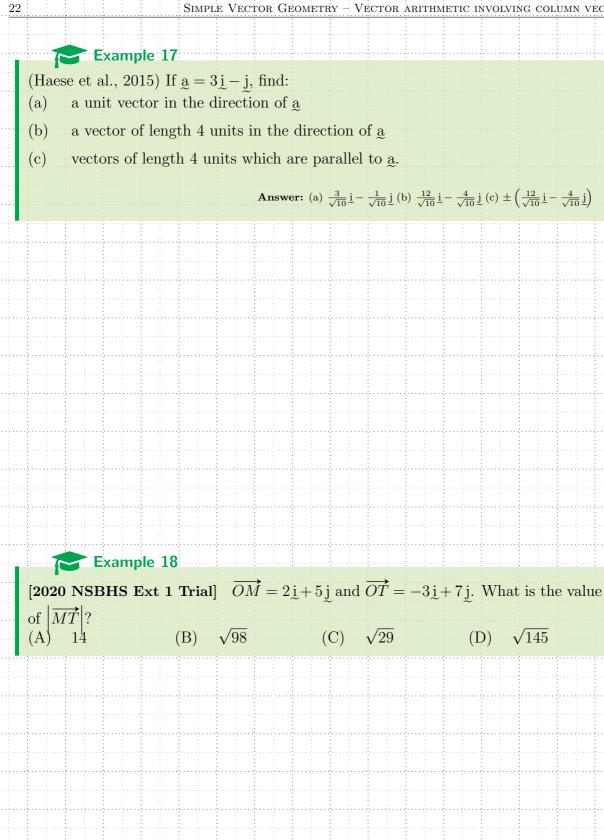


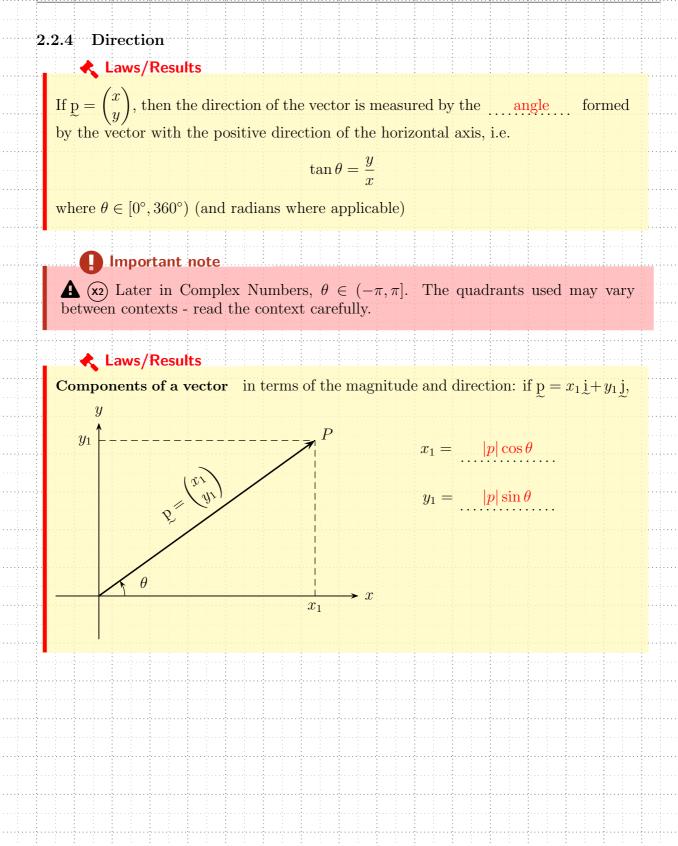
2.2 Vector arithmetic involving column vectors

2.2.1 Addition Laws/Results If $\underline{\mathbf{p}} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\underline{\mathbf{q}} = \begin{pmatrix} u \\ v \end{pmatrix}$, then $\underbrace{\mathbf{p}}_{\widetilde{\mathbf{n}}} \pm \underbrace{\mathbf{q}}_{\widetilde{\mathbf{n}}} = \begin{pmatrix} x \pm u \\ y \pm v \end{pmatrix}$ $= (x \pm u)\mathbf{j} + (y \pm v)\mathbf{j}$ Important note • Add <u>horizontal</u> components $(x \text{ direction}, \underline{i})$ with horizontal components • Add <u>vertical</u> components $(y \text{ direction}, \underline{j})$ with <u>vertical</u> components C GeoGebra Vector component addition Example 12 (Pender et al., 2019) (a) Given $\underline{u} = 2\underline{i} - \underline{j}$ and $\underline{v} = -\underline{i} + 2\underline{j}$, find: 1. i. 2u ii. iii. 3v2u + 3v(b) Draw all five vectors as position vectors. What figure do the origin and heads of 2u, 3v and 2u + 3v form? (a) Given $\underline{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, find $\underline{v} - \underline{u}$. 2. (b) Draw $\underline{u} = \overrightarrow{OU}$ and $\underline{v} = \overrightarrow{OV}$ as position vectors, and mark $\underline{v} - \underline{u}$.







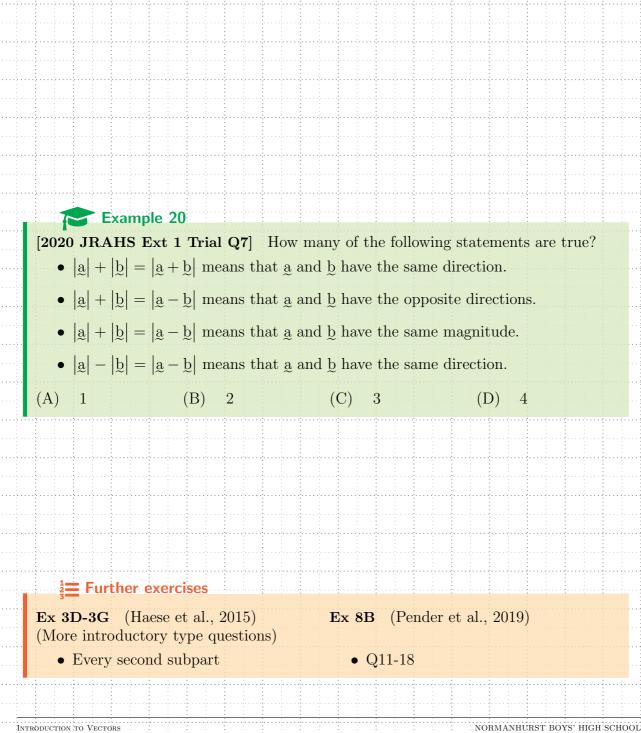


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(Pender et al., 2019)

- (a) Find the vector \underline{u} of length 10, with angle 150° to the positive direction of the horizontal axis.
- (b) Find the length and angle (exact length, angle to the nearest degree) of $\underline{y} = -\underline{i} 2\underline{j}$.

Answer: (a) $\underline{u} = -5\sqrt{3}\underline{i} + 5\underline{j}(b) |\underline{u}| = \sqrt{5}, \theta \approx 243^{\circ}.$



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Section 3

Vector Geometry with the Dot Product

3.1 **Definitions**

Definition 13

The dot product between two vectors $\underline{p} = x_1 \underline{i} + y_1 \underline{j}$ and $\underline{q} = x_2 \underline{i} + y_2 \underline{j}$ (also known as scalar product):

 $\underbrace{\mathbf{p}}_{\widetilde{\mathbf{q}}} \cdot \underbrace{\mathbf{q}}_{\widetilde{\mathbf{q}}} = \dots x_1 x_2 + y_1 y_2$

GeoGebra Dot Product Insight

Example 21

Use $\underline{p} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$ etc to prove the following: (a) $\underline{p} \cdot \underline{q} = \underline{q} \cdot \underline{p}$ (Commutativity) (b) $\underline{p} \cdot \underline{p} = \left| \underline{p} \right|^2$

(c)
$$\underline{p} \cdot (\underline{q} + \underline{r}) = \underline{p} \cdot \underline{q} + \underline{p} \cdot \underline{r}$$
 (Associativity)

(d)
$$\left(\underline{p} + \underline{q}\right) \cdot \left(\underline{r} + \underline{s}\right) = \underline{p} \cdot \underline{r} + \underline{p} \cdot \underline{s} + \underline{q} \cdot \underline{r} + \underline{q} \cdot \underline{s}$$



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(Haese et al., 2015, Ex 3I) Explain why $\underline{a} \cdot \underline{b} \cdot \underline{c}$ is meaningless.

3.2 Angle between two vectors

The dot product enables the angle θ between two non-zero vectors to be found. Definition 14

b The angle θ between two non-zero vectors **p** and **q** is related by:

$$\therefore \underbrace{\mathbf{p}}_{\widetilde{\mathbf{x}}} \cdot \underbrace{\mathbf{q}}_{\widetilde{\mathbf{x}}} = \left| \underbrace{\mathbf{p}}_{\widetilde{\mathbf{x}}} \right| \left| \underbrace{\mathbf{q}}_{\widetilde{\mathbf{x}}} \right| \cos \theta$$

📰 Steps

- Proof
- 1. Draw situation representing the angle θ between two vectors \underline{p} and \underline{q} , and vector $\underline{q} \underline{p}$.

2. Apply the cosine rule to the triangle:

$$\left|\underline{\mathbf{q}} - \underline{\mathbf{p}}\right|^2 = \left|\underline{\mathbf{p}}\right|^2 + \left|\underline{\mathbf{q}}\right|^2 - 2\left|\underline{\mathbf{p}}\right|\left|\underline{\mathbf{q}}\right|\cos\theta$$

3. Let $\underline{p} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$ and $\underline{q} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$ and replace magnitudes with components of \underline{p} and \underline{q} :

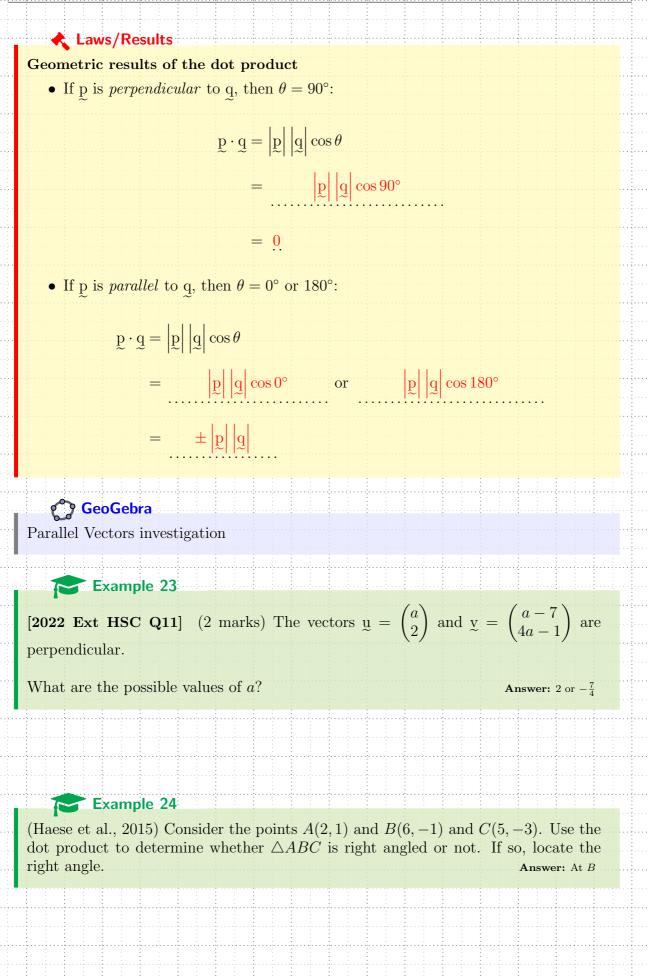
$$\therefore \underbrace{\mathbf{p}}_{\approx} \cdot \underbrace{\mathbf{q}}_{\approx} = |\underbrace{\mathbf{p}}_{\approx}| |\underbrace{\mathbf{q}}_{\approx}| \cos \theta$$

4. Consequently,

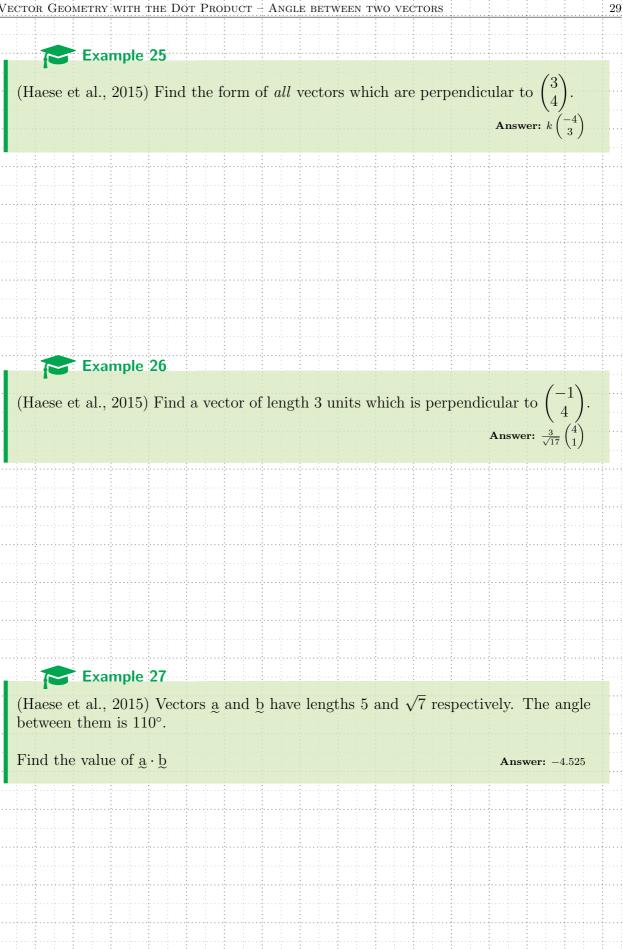
$$\cos \theta =$$

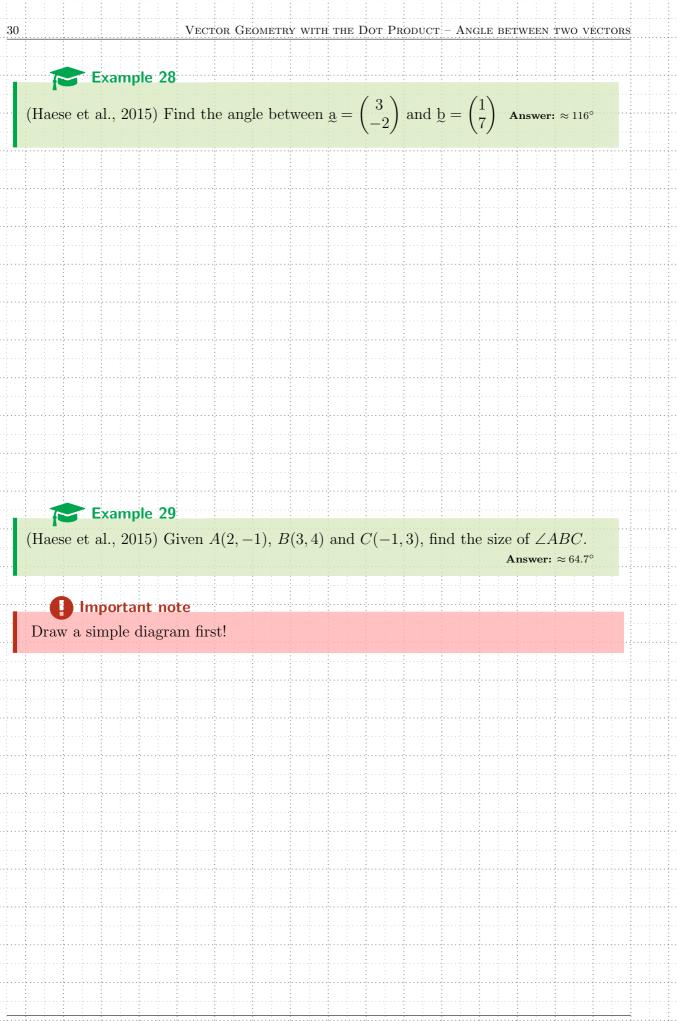
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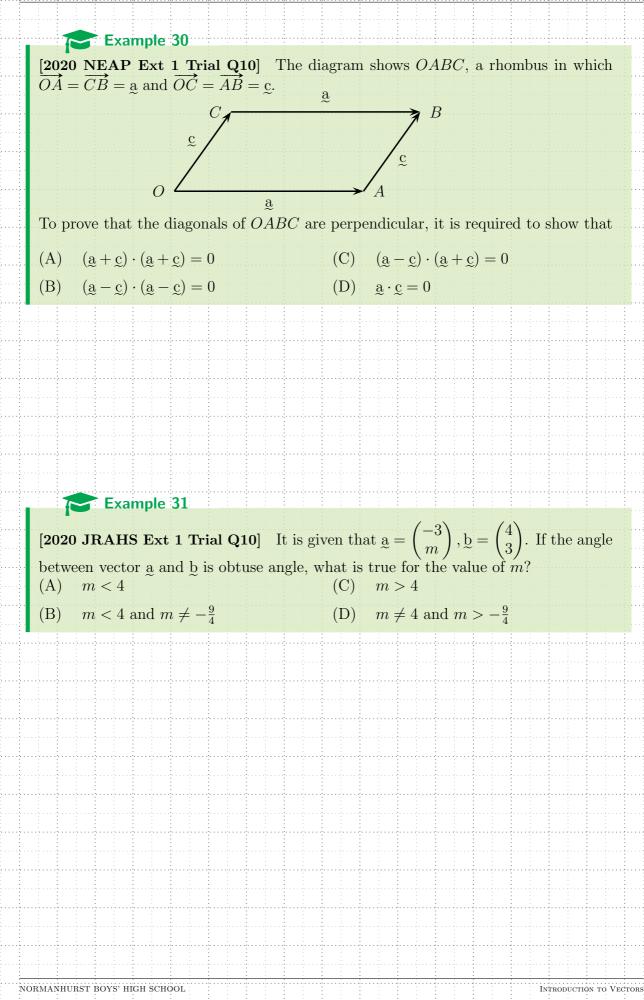
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INTRODUCTION TO VECTORS





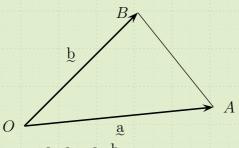


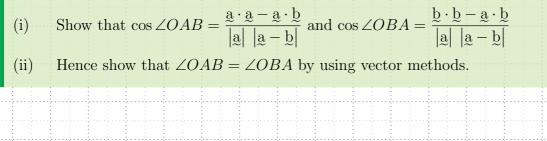
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Example 32

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[2020 Independent Ext 1 Trial Q11] In the diagram, OAB is an acute angled triangle in which OA = OB. The vector $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$.





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[2020 JRAHS Ext 1 Trial Q12] In the isosceles triangle ABC , $\left \overrightarrow{AB}\right = \left \overrightarrow{AC}\right $.														. I)															
is the midpoint of side AB and E is the midpoint of side AC . \overrightarrow{CD} is perpendicula																														
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Evertises

Ex 3I-3J (Haese et al., 2015) (More introductory type questions) **Ex 8C** (Pender et al., 2019)

• Every second subpart

• All questions

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3.3 Other geometric proofs

Important note

Geometric proofs involving vectors are vastly different to the Euclidean Geometry proofs!

📃 Steps

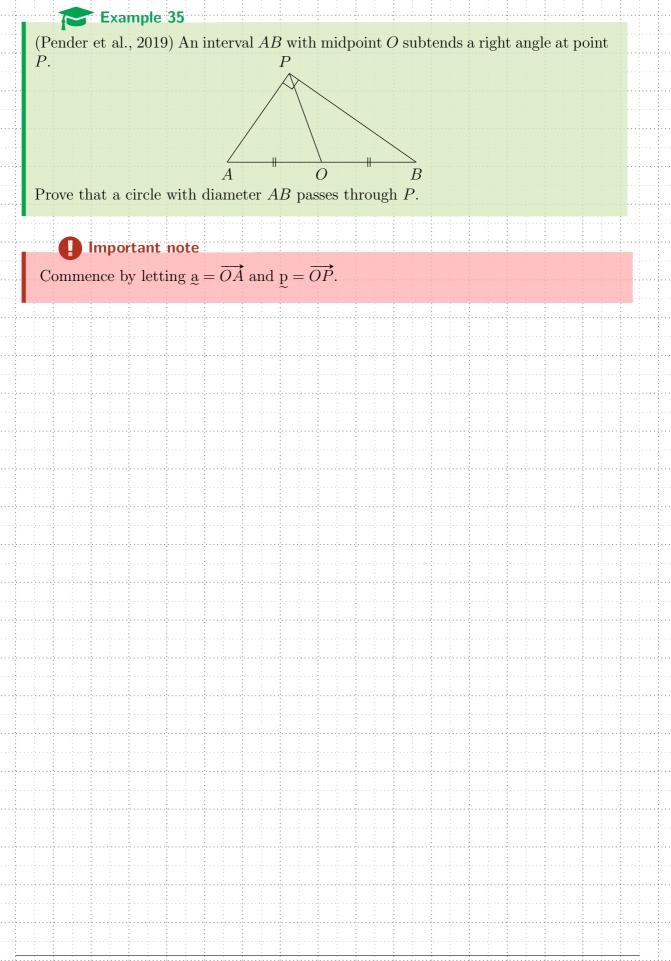
Use the following as a guide to these proofs:

- Draw picture
- Introduce vectors, by choosing one of the vertices or a point outside the figure as a reference point/origin.
- Use the <u>sum</u> and <u>scalar</u> <u>multiple</u> of vectors to complete a triangle.
- <u>Parallel</u> vectors are <u>scalar</u> <u>multiples</u> of each other.
- The dot product is zero when two non zero vectors are at right angles
- The <u>dot</u> product may also allow access to the angle between some of the vectors.

Example 34

[Ex 8D Q2] **66** P, Q, R and S are midpoints of AB, BC, CD and DA respectively. Let $\overrightarrow{AB} = \underline{a}, \overrightarrow{BC} = \underline{b}$ and $\overrightarrow{AD} = \underline{d}$ and $\overrightarrow{DC} = \underline{c}$.

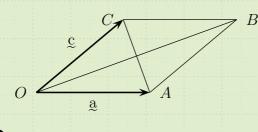
- (a) Explain why $\underline{a} + \underline{b} = \underline{d} + \underline{c}$.
- (b) Express \overrightarrow{PQ} in terms of \underline{a} and \underline{b} .
- (c) Express \overrightarrow{SR} in terms of \underline{d} and \underline{c} .
- (d) Hence show that $\overrightarrow{PQ} = \overrightarrow{SR}$.
- (e) Deduce that the quadrilateral PQRS is a parallelogram.



Example 36

36

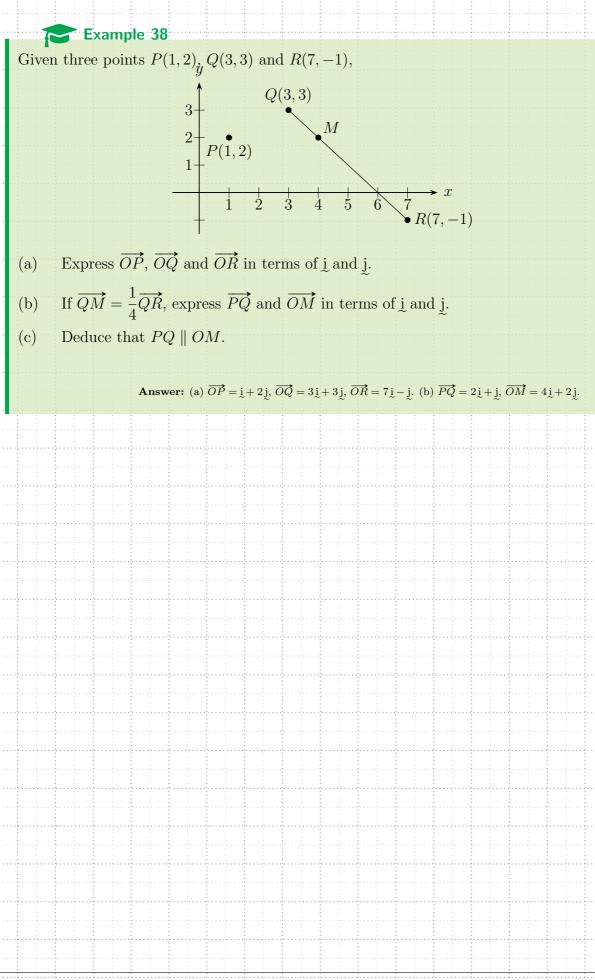
[Ex 8D Q6] **66** In the diagram OABC is a parallelogram whose diagonals OB and AC are equal. The points A and C have respective position vectors \underline{a} and \underline{c} relative to O.



- (a) Explain why $\overrightarrow{CB} = \underline{a}$.
- (b) Write \overrightarrow{AC} in terms of \underline{c} and \underline{a} .
- (c) Explain why $|\underline{c} + \underline{a}| = |\underline{c} \underline{a}|$.
- (d) Use the result in the previous part, and the fact that $|\underline{y}|^2 = \underline{y} \cdot \underline{y}$, to show that $\underline{a} \cdot \underline{c} = 0$.
- (e) What conclusions can be made about a parallelogram whose diagonals are equal?

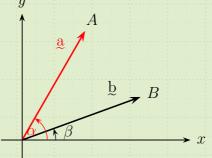
Example 37

[Ex 8D Q10] **66** Use vectors to prove that the sum of the squares of the lengths of the two diagonals of a parallelogram is equal to the sum of the squares of the lengths of the four sides.



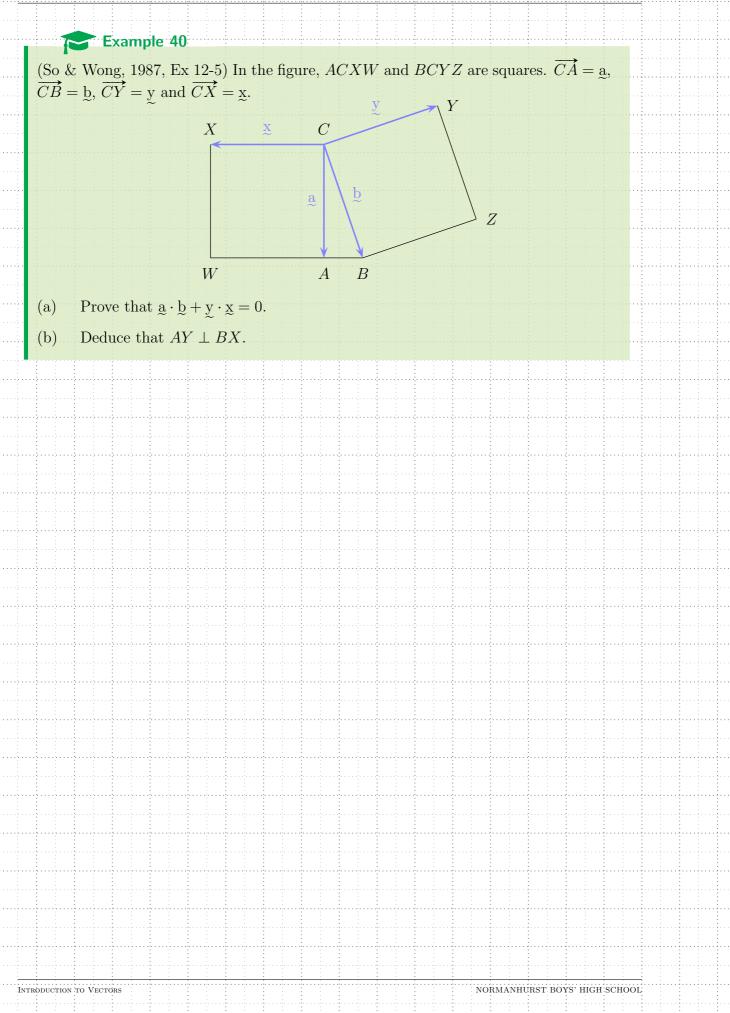
Example 39

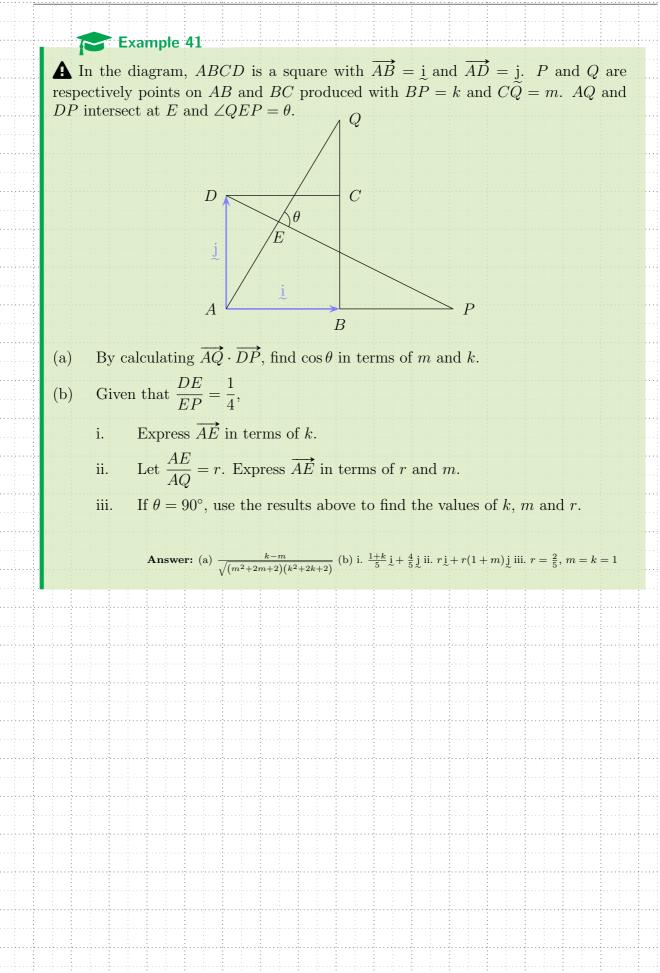
In the following figure, the position vectors of the points A and B with respect to the origin are \underline{a} and \underline{b} respectively. They are unit vectors making angles α and β with the positive direction of the x axis.



Prove that:

- (a) $\underline{a} = \cos \alpha \underline{i} + \sin \alpha \underline{j}$ and $\underline{b} = \cos \beta \underline{i} + \sin \beta \underline{j}$
- (b) $\cos(\alpha \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta.$





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INTRODUCTION TO VECTORS

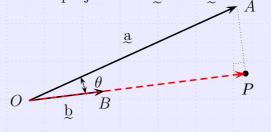
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3.4 Projection of a vector on to another vector

The *projection* of a vector \underline{a} on another vector \underline{b} is a vector parallel to \underline{b} with the following notation:

proj_b <u>a</u>

In the diagram given, \overrightarrow{OP} is the projection of a on to b.



Assumption: \underline{b} is a <u>non</u> <u>zero</u> <u>vector</u>.

Important note

Plain English:the projection of vector \underline{a} on to vector \underline{b} is the ________.cast'from \underline{a} on to \underline{b} .

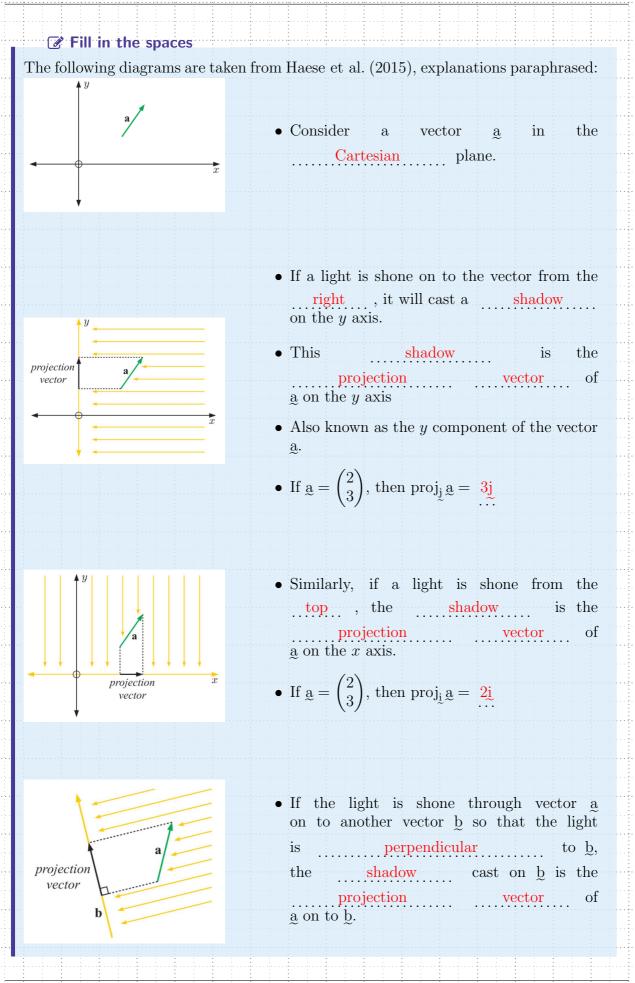
Laws/Results

The projection of \underline{a} on to \underline{b} , $\underline{b} \neq \underline{0}$:

$$\operatorname{proj}_{\underline{b}} \underline{\mathbf{a}} = \left(\left(\frac{\underline{\mathbf{a}} \cdot \underline{\mathbf{b}}}{\left| \underline{\mathbf{b}} \right|^2} \right) \underline{\mathbf{b}} \right)$$

C GeoGebra

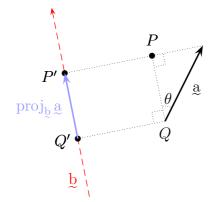
Projection of vector \underline{a} on to \underline{b}





3.4.1 Derivation

Consider the diagram:



- Suppose θ is the acute angle between \underline{a} and \underline{b} .
- Give the relationship between PQ, |a| and θ :

$$\frac{PQ}{|\underline{a}|} = \dots \cos \theta$$
$$\therefore PQ = |\underline{a}| \cos \theta$$

• As $\operatorname{proj}_{\underline{b}} \underline{a}$ is a vector in the same direction to \underline{b} , it is also the unit vector in \underline{b} 's direction, multiplied by a scalar multiple:*

$$\operatorname{proj}_{\underline{b}} \underline{a} = \underbrace{PQ}_{\text{scalar multiple, 'magnitude'}} \widehat{\underline{b}}$$

$$= \underbrace{|\underline{a} \cos \theta|}_{\text{scalar multiple, 'magnitude'}} \widehat{\underline{b}}$$

$$= \underbrace{|\underline{a}| |\cos \theta|}_{|\underline{b}|} \underbrace{|\underline{b}|}$$

$$(\ddagger)$$

• Recall also, that $\underline{\mathbf{a}} \cdot \underline{\mathbf{b}} = |\underline{\mathbf{a}}| |\underline{\mathbf{b}}| \cos \theta$, rearranging:

$$\cos \theta = \frac{\underline{\mathbf{a}} \cdot \underline{\mathbf{b}}}{|\underline{\mathbf{a}}| |\underline{\mathbf{b}}|}$$

Substitute in the result from (\ddagger) :

$$\operatorname{proj}_{\underline{b}} \underline{a} = \underbrace{|\underline{a} \cdot \underline{b}|}_{\underline{a} \cdot \underline{b}} |\underline{b}| |\underline{b}|}_{\underline{b} \cdot \underline{b}} = \frac{\underline{a} \cdot \underline{b}}{|\underline{b}|^2} \underline{b}$$

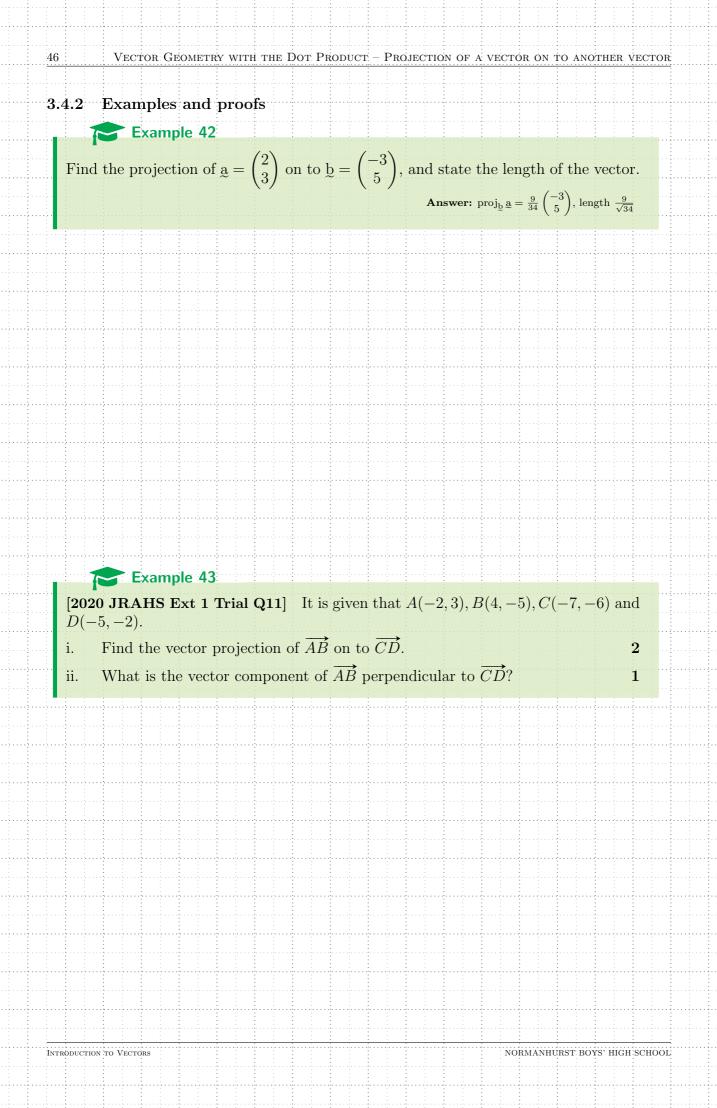
Laws/Results

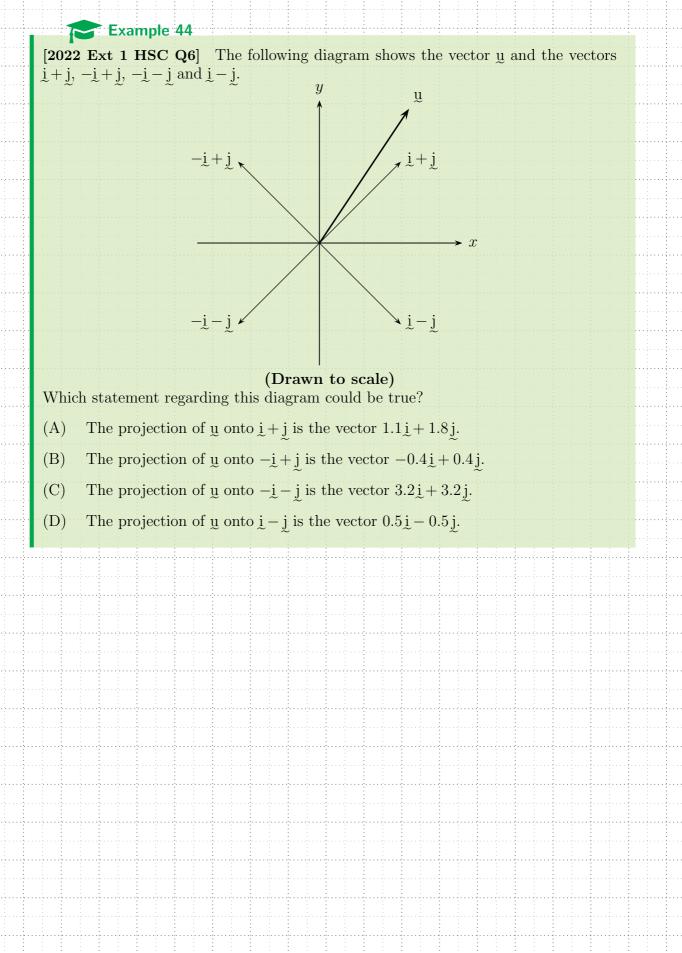
The projection of vector \underline{a} on to vector \underline{b} :

$$\operatorname{proj}_{\underline{b}} \underline{a} = \frac{\underline{a} \cdot \underline{b}}{|\underline{b}|^2} \underline{b} = \frac{\underline{a} \cdot \underline{b}}{\underline{b} \cdot \underline{b}} \underline{b}$$

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^{*}See Section 2.1.1 on page 13 $\,$





Example 45

[2022 Ext 1 HSC Q14] (3 marks) The vectors \underline{u} and \underline{v} are not parallel. The vector p is the projection of \underline{u} onto the vector \underline{v} .

The vector \underline{p} is parallel to \underline{v} so it can be written as $\lambda_0 \underline{v}$ for some real number λ_0 . (Do NOT prove this).

Prove that $|\underline{\mathbf{u}} - \lambda \underline{\mathbf{v}}|$ is smallest when $\lambda = \lambda_0$ by showing that, for all real numbers λ , $|\underline{\mathbf{u}} - \lambda_0 \underline{\mathbf{v}}| \leq |\underline{\mathbf{u}} - \lambda \underline{\mathbf{v}}|$.



Ex 3K (Haese et al., 2015) (More introductory type questions)

Ex 8E (Pender et al., 2019)

• Every second subpart

INTRODUCTION TO VECTORS

• All questions

Section A

Past examination questions

- Most questions in this section originate from various VCEs.
- Two additional terms which are not used in the NSW Syllabuses but have equivalents:

Definition 16

Vector resolute is synonymous with the the vector projection.

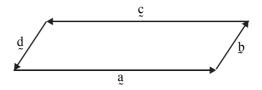
Definition 17

Scalar projection is the length of the vector projection, with a negative sign if the projection has an opposite direction with respect to \underline{b}

A.1 2006 VCE Specialist Mathematics

A.1.1 Paper 2 Section 1

15. In the parallelogram shown, $|\underline{a}| = 2 |\underline{b}|$.



1

Which one of the following statements is true?

- (A) $\underline{a} = 2\underline{b}$ (C) $\underline{b} \underline{d} = \underline{0}$ (E) $\underline{a} \underline{b} = \underline{c} \underline{d}$
- (B) $\underline{a} + \underline{b} = \underline{c} + \underline{d}$ (D) $\underline{a} + \underline{c} = \underline{0}$

A.1.2 Paper 2 Section 2

Question 8

Point A has position vector $\underline{a} = -\underline{i} - 4\underline{j}$, point B has position vector $\underline{b} = 2\underline{i} - 5\underline{j}$, point C has position vector $\underline{c} = 5\underline{i} - 4\underline{j}$, and point D has position vector $\underline{d} = 2\underline{i} + 5\underline{j}$ relative to the origin O.

(a) Show that \overrightarrow{AC} and \overrightarrow{BD} are perpendicular.

 $\mathbf{2}$

- (b) Use a vector method to find the cosine of $\angle ADC$, the angle between \overrightarrow{DA} and $2\overrightarrow{DC}$.
- (c) Find the cosine of $\angle ABC$, and hence show that $\angle ADC$ and $\angle ABC$ are **3** supplementary.

Point P has position vector $\mathbf{p} = 2\mathbf{i}$.

(d) Use the cosine of $\angle APC$ and an appropriate trigonometric formula to prove **3** that $\angle APC = 2\angle ADC$.

A.2 2008 VCE Specialist Mathematics

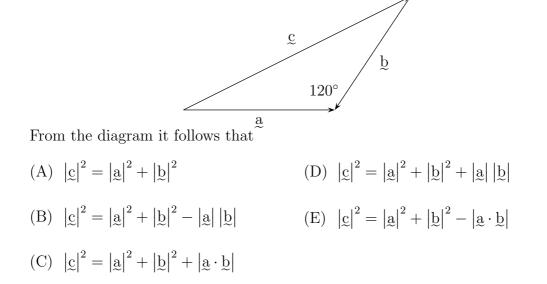
A.2.1 Paper 2 Section 1

- 17. If P, Q and R are three collinear points with position vectors $\underline{p}, \underline{q}$ and \underline{r} 1 respectively, where Q lies between P and R. If $\left|\overrightarrow{QR}\right| = \frac{1}{2} \left|\overrightarrow{PQ}\right|$, then \underline{r} is equal to
 - (A) $\frac{3}{2} \underbrace{q}_{\cdot} \frac{1}{2} \underbrace{p}_{\cdot}$ (C) $\frac{3}{2} \underbrace{q}_{\cdot} \frac{3}{2} \underbrace{p}_{\cdot}$ (E) $\frac{3}{2} \underbrace{p}_{\cdot} \frac{3}{2} \underbrace{q}_{\cdot}$
 - (B) $\frac{3}{2} \underbrace{\mathbf{p}}_{\sim} \frac{1}{2} \underbrace{\mathbf{q}}_{\sim}$ (D) $\frac{1}{2} \underbrace{\mathbf{p}}_{\sim} \frac{3}{2} \underbrace{\mathbf{q}}_{\sim}$

A.3 2009 VCE Specialist Mathematics

A.3.1 Paper 2 Section 1

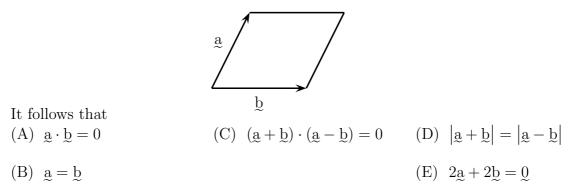
17. Vectors a, b and c are shown below.



A.4 2011 VCE Specialist Mathematics

A.4.1 Paper 2 Section 1

10. The diagram below shows a rhombus, spanned by the two vectors a and b.



A.5 2018 VCE Specialist Mathematics

A.5.1 Paper 2 Section 1

- 11. Consider the vectors given by $\underline{a} = m\underline{i} + \underline{j}$ and $\underline{b} = \underline{i} + m\underline{j}$, where $m \in \mathbb{R}$. If the **1** acute angle between \underline{a} and \underline{b} is 30°, then m equals
 - (A) $\sqrt{2} \pm 1$ (C) $\sqrt{3}, \frac{1}{\sqrt{3}}$ (E) $\frac{\sqrt{39}}{13}$ (B) $2 \pm \sqrt{3}$ (D) $\frac{\sqrt{3}}{4 - \sqrt{3}}$

1

- 12. If $|\underline{a} + \underline{b}| = |\underline{a}| + |\underline{b}|$ and $\underline{a}, \underline{b} \neq 0$, which one of the following is **necessarily** 1 true?
 - (A) <u>a</u> is parallel to <u>b</u> (B) $|\underline{a}| = |\underline{b}|$ (C) <u>a</u> = <u>b</u> (D) <u>a</u> = -<u>b</u> (E) <u>a</u> is perpendicular to <u>b</u>

A.6 2020 Ext 1 HSC

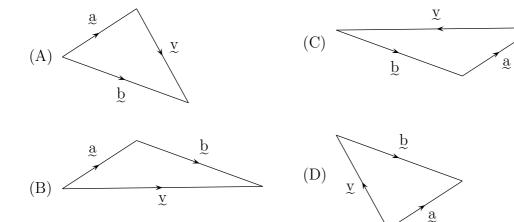
4. Maria starts at the origin and walks along all of the vector $2\underline{i} + 3\underline{j}$, then walks along all of the vector $3\underline{i} - 2\underline{j}$ and finally along all of the vector $4\underline{i} - 3\underline{j}$.

How far from the origin is she?

- (A) $\sqrt{77}$ (C) $2\sqrt{13} + \sqrt{5}$ (B) $\sqrt{85}$ (D) $\sqrt{5} + \sqrt{7} + \sqrt{13}$
- **6.** The vectors \underline{a} and \underline{b} are shown.



Which diagram below shows the vector $\underline{v} = \underline{a} - \underline{b}$?



9. The projection of the vector $\begin{pmatrix} 6\\7 \end{pmatrix}$ on to the line y = 2x is $\begin{pmatrix} 4\\8 \end{pmatrix}$. The point (6,7) is reflected in the line y = 2x to a point A.

What is the position vector of the point A?

(A) $\begin{pmatrix} 6\\12 \end{pmatrix}$ (B) $\begin{pmatrix} 2\\9 \end{pmatrix}$ (C) $\begin{pmatrix} -6\\7 \end{pmatrix}$ (D) $\begin{pmatrix} -2\\1 \end{pmatrix}$

Question 11

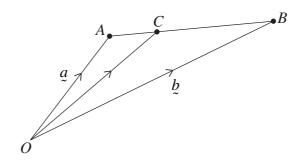
(b) For what value(s) of a are the vectors
$$\begin{pmatrix} a \\ -1 \end{pmatrix}$$
 and $\begin{pmatrix} 2a-3 \\ 2 \end{pmatrix}$ perpendicular? **3**

A.7 2020 Ext 2 HSC

This question does not contain any Extension 2 specific content, and so is placed in this summary instead.

Question 15

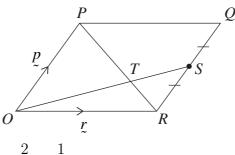
(b) The point C divides the interval AB so that $\frac{CB}{AC} = \frac{m}{n}$. The position vectors of A and B are a and b respectively, as shown in the diagram.



i. Show that
$$\overrightarrow{AC} = \frac{n}{m+n} (\underline{b} - \underline{a}).$$

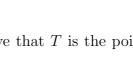
ii. Prove that
$$\overrightarrow{OC} = \frac{m}{m+n}\mathbf{a} + \frac{n}{m+n}\mathbf{b}$$

Let OPQR be a parallelogram with $\overrightarrow{OP} = \underbrace{p}_{i}$ and $\overrightarrow{OR} = \underbrace{r}_{i}$. The point S is the midpoint of QR and T is the intersection of PR And OS, as shown in the diagram.



iii. Show that
$$\overrightarrow{OT} = \frac{2}{3}\overrightarrow{r} + \frac{1}{3}\overrightarrow{p}$$
.

iv. Using parts (ii) and (iii), or otherwise, prove that T is the point that **1** divides the interval PR in the ratio 2:1.



 $\mathbf{2}$

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A.8 2021 Ext 1 HSC

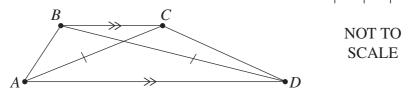
Question 14

(a) A plane needs to travel to a destination that is on a bearing of 063°. The engine is set to fly at a constant 175 km/h. However, there is a wind from the south with a constant speed of 42 km/h.

On what constant bearing, to the nearest degree, should the direction of the plane be set in order to reach the destination? Answer: 075°

(c) i. For vector
$$\underline{\mathbf{y}}$$
, show that $\underline{\mathbf{y}} \cdot \underline{\mathbf{y}} = |\underline{\mathbf{y}}|^2$.

ii. In the trapezium ABCD, BC is parallel to AD and $\left|\overrightarrow{AC}\right| = \left|\overrightarrow{BD}\right|$.



Let
$$\underline{a} = \overrightarrow{AB}$$
, $\underline{b} = \overrightarrow{BC}$ and $\overrightarrow{AD} = k\overrightarrow{BC}$, where $k > 0$

Using part (i) or otherwise, show $2\underline{\mathbf{a}} \cdot \underline{\mathbf{b}} + (1-k) \left|\underline{\mathbf{b}}\right|^2 = 0.$

A.9 2022 Ext 1 HSC

(B) $0 \le \theta < \frac{2\pi}{3}$

(C) $\frac{\pi}{3} < \theta \le \pi$

8. The angle between two unit vectors \underline{a} and \underline{b} is θ and $|\underline{a} + \underline{b}| < 1$.

Which of the following best describes the possible range of values of θ ?

- (A) $0 \le \theta < \frac{\pi}{3}$ (D) $\frac{2\pi}{3} < \theta \le \pi$
 - **Note:** attempt this after Topic 9 (Radians) has been completed.

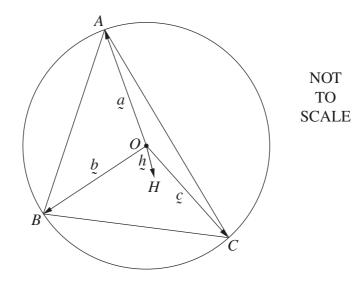
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3

Question 13

(a) Three different points A, B and C are chosen on a circle centred at O.

Let $\underline{\mathbf{a}} = \overrightarrow{OA}$, $\underline{\mathbf{b}} = \overrightarrow{OB}$ and $\underline{\mathbf{c}} = \overrightarrow{OC}$. Let $\underline{\mathbf{h}} = \underline{\mathbf{a}} + \underline{\mathbf{b}} + \underline{\mathbf{c}}$ and let H be the point such that $\overrightarrow{OH} = \underline{\mathbf{h}}$, as shown in the diagram.



Show that \overrightarrow{BH} and \overrightarrow{CA} are perpendicular.

A.10 2023 Ext 1 HSC

- 6. Given two non-zero vectors \underline{a} and \underline{b} , let \underline{c} be the projection of \underline{a} onto \underline{b} . What is the projection of $10\underline{a}$ onto $2\underline{b}$?
 - (A) $2\underline{c}$ (B) $5\underline{c}$ (C) $10\underline{c}$ (D) $20\underline{c}$

3

 $\mathbf{1}$

3

4

Question 14

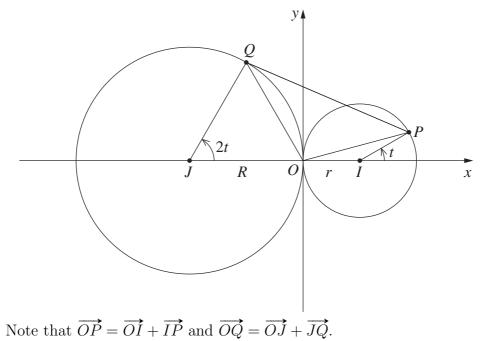
(c) i. Given a non-zero vector $\begin{pmatrix} p \\ q \end{pmatrix}$, it is known that the vector $\begin{pmatrix} q \\ -p \end{pmatrix}$ is perpendicular to $\begin{pmatrix} p \\ q \end{pmatrix}$ and has the same magnitude. (Do NOT prove this).

Points A and B have position vectors $\overrightarrow{OA} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, respectively.

Using the given information, or otherwise, show that the area of $\triangle OAB$ is $\frac{1}{2} |a_1b_2 - a_2b_1|$.

ii. The point P lies on the circle centred at I(r, 0) with radius r > 0, such that \overrightarrow{IP} makes an angle of t to the horizontal.

The point Q lies on the circle centred at J(-R, 0) with radius 2t to the horizontal.

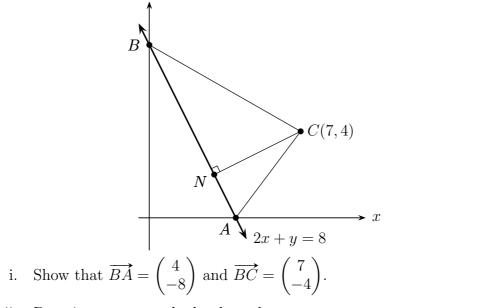


Using part (i), or otherwise, find the values of t, where $-\pi \leq t \leq \pi$, that maximise the area of $\triangle OPQ$.

2024 NBHS Assessment Task 1 A.11

Question 4

(a)The diagram shows $\triangle ABC$, and the line 2x + y = 8 with x and y intercepts at A and B respectively. The point C has coordinates (7, 4) and N is a point on AB such that $CN \perp AB$.



By using vector methods, show that ii.

$$\cos \angle ABC = \frac{3}{\sqrt{13}}$$

Find a vector $\underline{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ such that \underline{u} is perpendicular to \overrightarrow{BA} . iii. 1 $\mathbf{2}$

iv. Hence show that

$$\overrightarrow{NC} = \begin{pmatrix} 4\\2 \end{pmatrix}$$

Use vector methods to find the coordinates of N. v.

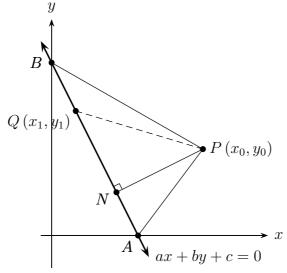
 $\mathbf{1}$

2

2

Question continues overleaf...

A line ℓ with equation ax + by + c = 0 is drawn in the Cartesian plane, and $P(x_0, y_0)$ is located off the line ℓ , and point Q has coordinates $Q(x_1, y_1)$ which lies on the line ℓ . N is a point that is located on the line ℓ such that $PN \perp AB$.



You are given that $\underline{\mathbf{y}} = \begin{pmatrix} a \\ b \end{pmatrix}$ is a vector that is perpendicular to \overrightarrow{BA} (Do NOT prove this).

vi. By finding \overrightarrow{QP} using the coordinates given, show that

$$\overrightarrow{\text{NP}} = \frac{ax_0 + by_0 + c}{\sqrt{a^2 + b^2}}\widehat{\mathfrak{Y}}$$

vii. Briefly explain why the shortest distance from a point (x_0, y_0) to a line ax + by + c = 0 is given by

$$\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

Answers

iii. Any scalar multiple of $\begin{pmatrix} 2\\1 \end{pmatrix}$

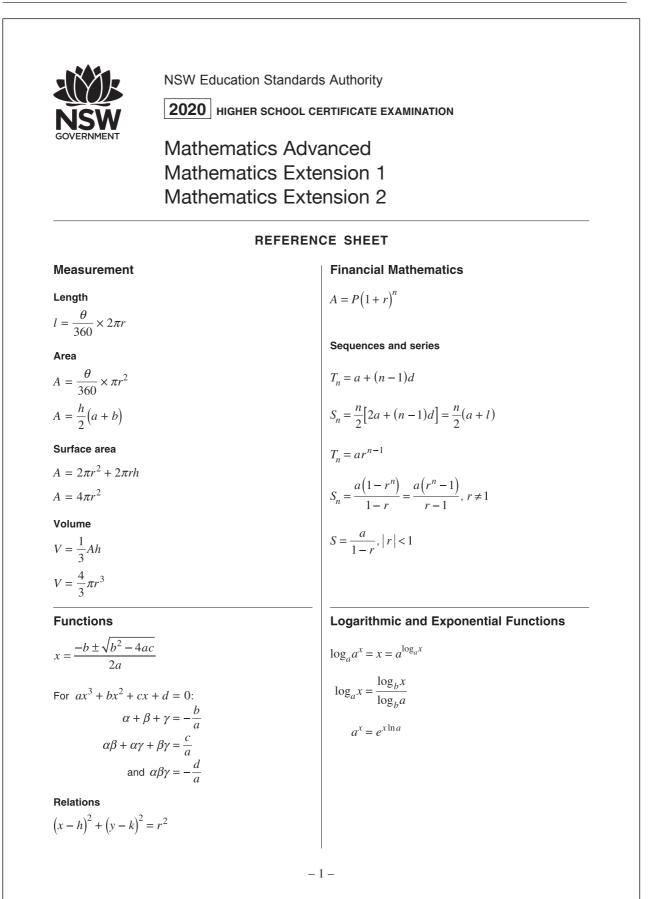
v. (3, 2)

vii.Show. Insufficient to state the perpendicular distance formula.

iv. Show

vi. Hint: use $ax_1 + by_1 + c = 0$

NESA Reference Sheet – calculus based courses



Trigonometric Functions

 $\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$ $A = \frac{1}{2}ab\sin C$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\frac{\sqrt{2}}{45^{\circ}}$ $C^{2} = a^{2} + b^{2} - 2ab\cos C$ $\cos C = \frac{a^{2} + b^{2} - c^{2}}{2ab}$ $l = r\theta$ $A = \frac{1}{2}r^{2}\theta$ $\frac{60^{\circ}}{1}$

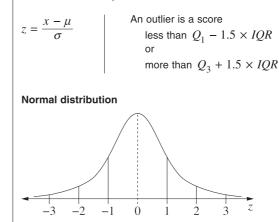
Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$
$$\csc A = \frac{1}{\sin A}, \ \sin A \neq 0$$
$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$
$$\cos^2 x + \sin^2 x = 1$$

Compound angles

 $\sin(A + B) = \sin A \cos B + \cos A \sin B$ $\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ If $t = \tan \frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$ $\cos A = \frac{1 - t^2}{1 + t^2}$ $\tan A = \frac{2t}{1 - t^2}$ $\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$ $\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$ $\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$ $\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$ $\sin^2 nx = \frac{1}{2} (1 - \cos 2nx)$ $\cos^2 nx = \frac{1}{2} (1 + \cos 2nx)$

Statistical Analysis



- approximately 68% of scores have z-scores between -1 and 1
- approximately 95% of scores have z-scores between -2 and 2
- approximately 99.7% of scores have z-scores between -3 and 3

$$E(X) = \mu$$

 $\sqrt{3}$

$$Var(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Continuous random variables

$$P(X \le x) = \int_{a}^{x} f(x) dx$$
$$P(a < X < b) = \int_{a}^{b} f(x) dx$$

Binomial distribution

$$P(X = r) = {^nC_r}p^r(1-p)^{n-r}$$

$$X \sim Bin(n, p)$$

$$\Rightarrow P(X = x)$$

$$= {n \choose x}p^x(1-p)^{n-x}, x = 0, 1, \dots, n$$

$$E(X) = np$$

$$Var(X) = np(1-p)$$

- 2 -

Differential Calculus		Integral Calculus					
Function	Derivative	$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$					
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$	$\int \frac{1}{n+1} \frac{1}{n+1} \frac{1}{n+1} \frac{1}{n+1}$ where $n \neq -1$					
y = uv	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$	$\int f'(x)\sin f(x)dx = -\cos f(x) + c$					
y = g(u) where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	$\int f'(x)\cos f(x)dx = \sin f(x) + c$					
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$					
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$	$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$					
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$						
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$	$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$					
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$	$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$					
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$					
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$	$\int f'(x) = 1 + f(x)$					
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$	$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$					
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$					
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$	$\int_{a}^{b} f(x) dx$					
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$	$\approx \frac{b-a}{2n} \Big\{ f(a) + f(b) + 2 \Big[f(x_1) + \dots + f(x_{n-1}) \Big] \Big\}$ where $a = x_0$ and $b = x_n$					
- 3 -							

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \dots + \binom{n}{r}x^{n-r}a^{r} + \dots + a^{n}$$

Vectors

$$\begin{split} \left| \begin{array}{c} \underline{u} \right| &= \left| \begin{array}{c} x\underline{i} + y\underline{j} \right| = \sqrt{x^2 + y^2} \\ \\ \underline{u} \cdot \underline{v} &= \left| \begin{array}{c} \underline{u} \right| \right| \underline{v} \left| \cos \theta = x_1 x_2 + y_1 y_2 \right|, \\ \\ \text{where } \begin{array}{c} \underline{u} &= x_1 \underline{i} + y_1 \underline{j} \\ \\ \\ \text{and } \begin{array}{c} \underline{v} &= x_2 \underline{i} + y_2 \underline{j} \end{array} \end{split}$$

 $r_{\tilde{u}} = a + \lambda b_{\tilde{u}}$

Complex Numbers

 $z = a + ib = r(\cos\theta + i\sin\theta)$ $= re^{i\theta}$ $\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$ $= r^n e^{in\theta}$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$
$$x = a\cos(nt + \alpha) + c$$
$$x = a\sin(nt + \alpha) + c$$
$$\ddot{x} = -n^2(x - c)$$

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